ASYMPTOTIC CURVE OF THE SCATTERING OF THE PRODUCTS OF A STEADY-STATE DETONATION

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We consider the steady-state problem of the outflow of the detonation products from the end of a flat or cylindrical charge of explosive into a vacuum. We neglect the curvature of the sonic surface (the Chapman–Jouguet surface) and shall consider it as flat up to the boundaries of the charge. Problems close in their statement were discussed in [1-3], both in a study of the phenomenon of the scattering of detonation products and with a discussion of questions of the throwing of plates and shells. The axisymmetric problem of the outflow of an ideal gas into a vacuum was solved in [4]. We give below the results of a calculation of the flow of detonation products, right up to considerable distances from the detonation front, where, as follows from the general considerations, it takes on a simple asymptotic character. The problem formulated has not been analyzed up to the present time with the aim of studying such asymptotic curves.

In a system of coordinates moving with the velocity of the detonation, the flow of the products is described by the system of equations

$$\begin{gathered}\partial v/\partial x - \partial u/\partial r = 0,\\ \frac{\partial}{\partial x} r^{\lambda}\rho u + \frac{\partial}{\partial r} r^{\lambda}\rho v = 0,\\ u^2 + v^2 + 2h(\rho) = U^2,\end{gathered} \tag{1}$$

where x and r are the longitudinal and transverse coordinates (Fig. 1); u and v are the corresponding projections of the velocity; ρ is the density; $h(\rho)$ is the enthalpy; U is the limiting velocity of the scattering; and $\lambda = 0$ in the plane case and $\lambda = 1$ in the axisymmetric case. The calculating region is shown in Fig. 1; AO is the sonic surface; AQ is the limiting line of the flow; and Ox is the plane (axis) of symmetry.

The flow is described using dimensionless quantities, so that the boundary conditions assume the form

$$\begin{gathered}u = 1,\ v = 0,\ \rho = 1 \ \text{for}\ \ x = 0,\ 0 \leqslant r < 1;\\ v = 0\ \text{for}\ \ x > 0,\ r = 0.\end{gathered} \tag{2}$$

The second condition flows out of the symmetry of the problem. In addition, in the neighborhood of the point x = 0, r = 1, there is Prandtl–Meyer flow. To complete the statement of the problem, the equation of state or the isentrope of the detonation products must be selected. Here

$$h(\rho) = \int dp/\rho,$$

where $p(\rho)$ is the pressure with constant entropy. In the first approximation, the detonation products expand in accordance with an isentropic law [5]

$$p(\rho) = \rho^{\gamma}/\gamma, \tag{3}$$

where γ = const = 3.2-3.4 for different explosives. However, this approximation is valid for sufficiently great pressures (on the order of several kilobars); with lower pressures, the value of γ decreases to γ = 1.35-1.25. A more exact isentrope of the discharge of real detonation products is given in [6] (Hexogen):

$$p(\rho) = \rho^{1.25}(a_0 + a_1\rho + a_2\rho^2 + a_3\rho^3), \tag{4}$$

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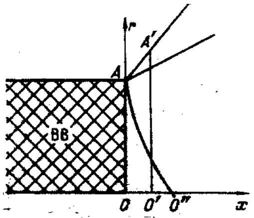

Fig. 1

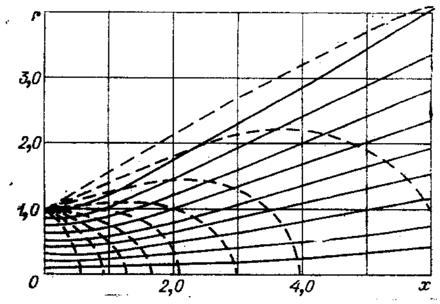

Fig. 2

TABLE 1

n	0	1	2	3	4	5	6	7
α	0,796616	—0,25	0,392498	—0,54775	0,80621	—1,188118	1,4	—
β	1	0,2	0,163333	0,150666	0,152766	0,180308	0,083108	0,036642

TABLE 2

Type of explosive	ρ_0, g/cm³	$\varkappa$	γ
Hexogen	1,66	0,023	1,25
Hexogen	1,01	0,057	1,25
Pentolite	1,65	0,005	1,35

where a_i ($i=0,\ldots,3$) are interpolation coefficients, and, in [7], the equation of state of pentolite is given:

$$p(\rho,\ E) = A\rho E + B\rho^4 + C\exp(-K/\rho), \tag{5}$$

where E is the internal energy; A, B, C, and K are constant quantities. The problem formulated was solved both using (3) and using Eqs. (4) and (5).

For solution of the problem posed, describing the flow of a gas with a supersonic velocity, the initial data, given at the sonic line, must be used to calculate the parameters of the flow in some neighborhood AO, belonging to the region under consideration. In the plane case, the solution given in [1] was used. In the axisymmetric case, the problem was solved in [4]; however, the use of a solution presented in the form of tables is difficult. A more convenient representation of the problem can be obtained, taking as independent variables

$$t = x/(1-r),\ y = 1-r,$$

and, as the sought functions, the quantities

$$\theta = \operatorname{arctg}(v/u),\ z = \ln\rho,\ \omega = \ln(u^2 + v^2)^{1/2}.$$

In these variables, the system (1) has the form ($\lambda = 1$)

$$\begin{gathered}(t - \operatorname{tg}\theta)\partial\omega/\partial t - y\partial\omega/\partial y - (1 + t)\partial\theta/\partial t + y\operatorname{tg}\theta\partial\theta/\partial y = 0;\\ y\operatorname{tg}\theta/(1-y) + (1 + t\operatorname{tg}\theta)\frac{\partial(z+\omega)}{\partial t} - y\frac{\partial}{\partial y}(z+\omega) + (t - \operatorname{tg}\theta)\,\partial\theta/\partial t - y\,\partial\theta/\partial y = 0,\\ \exp(2\omega) + 2h(\exp(z)) = U^2.\end{gathered} \tag{6}$$

For the equation of state (3), taking account of the insignificant change of the density of the detonation products in the region under consideration, the latter equation assumes the form

$$\exp(2\omega) = 1/2\ln((\gamma+1)/(\gamma-1) - 2\exp((\gamma-1)z)/(\gamma-1)),$$

where the last equation is transformed taking account of (3). The boundary conditions assume the form

$$\begin{array}{ll}\omega=0,\ \ z=0,\ \ \theta=0 & \text{for }\ t=0,\ \ 0<y\leqslant 1;\\ \theta=0 & \text{for }\ t\geqslant 0,\ \ y=1;\\ \omega=t^2/(\gamma+1)+0(t^3) & \text{for }\ y=0.\end{array} \tag{7}$$

The latter equation is an expansion in series of a solution describing Prandtl – Meyer flow. We shall seek the solution in the form of the series

$$\theta = \sum_{n=1}^{\infty} F_n(y)\, t^n, \quad \omega = \sum_{n=1}^{\infty} Q_n(y)\, t^n, \quad z = \sum_{n=1}^{\infty} S_n(y)\, t^n.$$

From (6), taking account of (7), limiting the discussion to the first three terms of the series, we obtain

$$F_1 = F_2 = 0;\ Q_1 = Q_3 = 0;\ S_1 = S_3 = 0;\ S_2 = -Q_2;$$
$$F_3 = (2/3)Q_2 - (y/3)dQ_2/dy.$$

It is more convenient to seek $Q_2(y)$ introducing the function $G = [6(\gamma - 1)/y^2]Q_2$, which is determined from the solution of the following boundary-value problem:

$$d^2G/dy^2 + 1/(y-1)\cdot dG/dy - G^2 = 0,$$
$$\lim_{y\to 0} y^2 G(y) = 6, \quad dG/dy = 0 \ \text{for} \ y = 1. \tag{8}$$

Problem (8) was solved numerically. The results of the calculations were approximated by polynomials of high power, which made it possible to operate with analytical, though cumbersome, expressions for the solution. For $Q_2(y)$ the following expressions were obtained:

$$Q_2(y) = 1/(\gamma+1) \sum_{n=0}^{7} \alpha_n y^n \quad \text{for} \quad 0 \leqslant y \leqslant 0.5; \quad Q_2(y) = y^2/[6(\gamma+1)] \sum_{n=0}^{6} \beta_n (1-y)^n \quad \text{for} \quad 0{,}5 \leqslant y \leqslant 1. \tag{9}$$

The interpolation coefficients α_n and β_n are given in Table 1. The maximal deviation of formulas (9) from the numerical solution does not exceed 0.5%.

The system of equations (1) with the boundary conditions (2) and the equations of state given was solved by the numerical method of characteristics. As the boundary between the calculating region and the vacuum there was taken a characteristic curve close to the limiting. The results of a calculation of the plane problem of the scattering of the detonation products of Hexogen [equation of state (4)] with an initial density $\rho_0 = 1.66$ g/cm^3 in terms of the line of the flow (solid lines, $\psi = 0.1, 0.2, \ldots, 0.9$) and the line of equal density (dashed lines, $\rho = 0.9, 0.8, \ldots, 0.1$) are given in Fig. 2. With their propagation over the detonation products, the discharge waves of the flow line become rectilinear, and the angle of inclination of each flow line to the Ox axis approaches a limiting constant value. In each concrete case, there exists a function describing the values of the limiting angles as a function of the flow line $\theta(\psi)$. With satisfaction of the well-known limitations on $\theta(\psi)$, the inverse function $\psi(\theta)$ can be constructed. This function will describe the flow of the detonation products for arbitrarily large values of x.

Let us now consider the flow parameters $q = (u^2 + v^2)^{1/2}$, $\theta = \arctan(v/u)$, and ρ in the polar coordinates $R = (x^2 + r^2)^{1/2}$ and $\varphi = \arctan(r/x)$. It is obvious that $\rho \to 0$ as $R \to \infty$; from the third equation of system (1) it follows that $q = q(\rho) \to U$ as $\rho \to 0$.

Since the outflow of the detonation products takes place into a region (a plane or solid angle) bounded by the limiting lines of the flow, while the flow lines, with a rise in R, become rectilinear, it is clear that $\theta(R, \varphi) \to \varphi$ as $R \to \infty$. For sufficiently large values of R, we represent the function ρ in the form of a power series;

$$\rho(R, \varphi) = \sum_{n=1}^{\infty} A_n(\varphi) R^{-n}. \tag{10}$$

The mass flow rate of the gas $\Delta\psi$ with R = const is written in the form

$$\Delta\psi = \rho(R, \varphi)\ q(\rho) \cos(\theta(R, \varphi) - \varphi)(2\pi \sin\varphi)^{\lambda} R^{1+\lambda} \Delta\varphi. \tag{11}$$

Passing to the limit $R \to \infty$, taking account of the above analysis, it can be established that the determining term in (10) has the number $1 + \lambda$ and is connected with ψ by the relationship

$$A_{1+\lambda} = \frac{1}{U} \lim \frac{1}{(2\pi \sin\varphi)^{\lambda}} \frac{\partial\psi}{\partial\varphi} = \frac{1}{U(2\pi\sin\varphi)^{\lambda}} \frac{d\psi_0}{d\varphi} = \frac{\mu_0(\varphi)}{U}.$$

Thus, with sufficiently large R, the scattering of the detonation products is described by the parameters

$$q(R, \varphi) = U, \ \theta(R, \varphi) = \varphi, \ \rho(R, \varphi) = \mu_0(\varphi)/(UR^{1+\lambda}). \tag{12}$$

A flow with such parameters is called asymptotic. Such a flow can be modelled by the outflow of a gas with a constant velocity along trajectories represented by rays going out from the origin of coordinates; the value of the mass flow rate of gas depends on the angular variable. The problem of the asymptotic of the scattering of

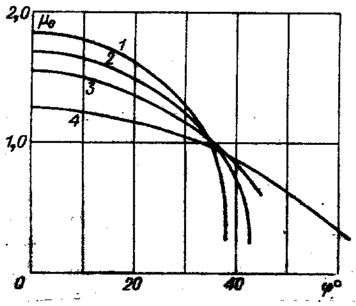

Fig. 3

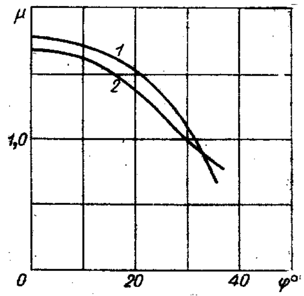

Fig. 4

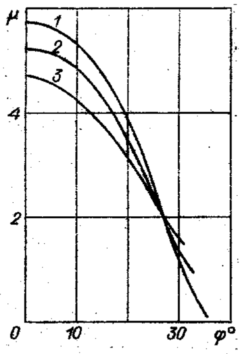

Fig. 5

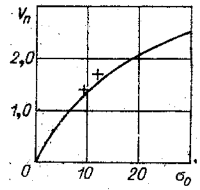

Fig. 6

TABLE 3

Gas under consideration	m_1	m_2	ν	Mass flow in calc. region, %	U	Rel. error of approx., %
Polytropic gas $\gamma = 3,0$	1,30	2,414	0,33	98	1,414	3,9
$\gamma = 2,8$	1,20	2,208	0,40	94	1,453	2,5
$\gamma = 2,7$	1,14	2,104	0,41	92	1,475	1,8
$\gamma = 2,4$	0,99	1,791	0,58	97	1,558	4,1
$\gamma = 2,0$	0,74	1,366	0,80	98	1,732	4
DP Hexogen, $\rho_0 = 1,01$ g/cm³	1,12	1,8	1,0	90	1,50	4
DP* Hexogen, $\rho_0 = 1,66$ g/cm³	0,86	1,4	1,0	85	1,59	4

*DP) Detonation product.

the products of a steady-state detonation reduces to a search for the function $\mu_0(\varphi)$ with respect to flow parameters calculated in a finite region. According to (12), this is equivalent to a search for the limiting function $\psi_0(\varphi)$. To solve the problem, we use the function $\psi(R, \varphi)$, i.e., the function $\psi(\varphi)$, found for some fixed finite value of R. Then

$$\mu(R, \varphi) = \frac{1}{(2\pi \sin \varphi)^{\lambda}} \frac{\partial \psi(R, \varphi)}{\partial \varphi},$$

and, using the latter relationship, from (12) and the equation of state (3)-(5), we may calculate the field of the pressure $p(R, \varphi)$. To simplify the subsequent computations, it is convenient to approximate (4) and (5) by an asymptotic formula:

$$p(\rho) = \varkappa \rho^{\gamma}, \tag{13}$$

$\rho \ll 1$; the effective values of $\varkappa$ and γ are given in Table 2. We now postulate that R is sufficiently great, and we find the change in the angle of inclination of the flow line $\Delta\theta$, brought about by the pressure $p(R, \varphi)$. For this we use an expression for the increment of the transverse component of the momentum, obtained from a calculation of the pressure gradient in this direction. Taking account of (12) and (13), this gives

$$\Delta\theta = -\frac{\varkappa}{2^{\lambda}(\gamma - 1)\,U^{(\gamma+1)}R^{(\gamma-1)(1+\lambda)}} \frac{1}{\mu(R, \varphi)} \frac{\partial}{\partial \varphi} \mu^{\gamma}(R, \varphi). \tag{14}$$

It is obvious that

$$\psi_0(\varphi) = \psi(R, \varphi - \Delta\theta). \tag{15}$$

From (15), taking account of (14), we can obtain

$$\Delta\mu(R,\varphi)=\mu_0(\varphi)-\mu(R,\varphi)=\frac{\varkappa}{2^{\lambda}(\gamma-1)U^{(\gamma+1)}\sin^{\lambda}\varphi}\frac{1}{R^{(1+\lambda)(\gamma-1)}}\frac{\partial}{\partial\varphi}\left[\sin^{\lambda}\varphi\frac{\partial}{\partial\varphi}\mu^{\gamma}(R,\varphi)\right].$$

Now let the functions $\mu(R_1, \varphi)$ and $\mu(R_2, \varphi)$ be calculated, where R_1 and R_2 are some sufficiently great values of R; then the limiting function $\mu_0(\varphi)$ is calculated using the formula

$$\mu_0(\varphi)=\frac{\mu(R_1,\varphi)R_1^{(\gamma-1)(1+\lambda)}-\mu(R_2,\varphi)R_2^{(\gamma-1)(1+\lambda)}}{R_1^{(\gamma-1)(1+\lambda)}-R_2^{(\gamma-1)(1+\lambda)}},$$

making it possible to use the parameters of the flow, found as a result of a numerical solution of the problem posed, to find the parameters of the asymptotic flow.

Curves of the functions $\mu_0(\varphi)$ for the plane case (i.e., $\lambda = 0$) are given in Fig. 3 ($\gamma = 3$, 2.7, 2.4, and 2 for curves 1-4, respectively) and Fig. 4 [1) the function $\mu(26, \varphi)$; 2) $\mu_0(\varphi)$ for the detonation products of Hexogen with $\rho = 1.66$ g/cm^3]. The calculations show that in the plane case the function $\mu_0(\varphi)/U$, in accordance with (12), determining the angular distribution of the density of the asymptotic flow, is approximated to a good degree of accuracy by the analytical dependence

$$\mu_0(\varphi)/U=m_1\cos^{\nu}(m_2\varphi), \tag{16}$$

where m_1, m_2, and ν are constants, determined from the calculation. The values of the constants for several variants of the problem are given in Table 3; the table also gives calculated values of U.

An example of solution of the axisymmetric problem is given in Fig. 5, where: 1) $\mu_0(\varphi)$ for the equation of state (3), $\gamma = 3$; 2,3) $\mu(10, \varphi)$ and $\mu_0(\varphi)$ for Hexogen (4) with $\rho_0 = 1.66$ g/cm^3.

As follows from the discussion, for solution of the problem posed, a calculation of the initial section of the supersonic flow according to parameters given at the sonic line is required. In [8], it is proposed to consider the flow of real detonation products, starting not from the sonic line, but from some plane at which $v = 0$, and $M > 1$, where M is the Mach number. A calculation of flows with such boundary data considerably simplifies the procedure of the solution of the problem. A numerical experiment was used to investigate the effect of the parameter ε (with assignment of the starting data in the form $v = 0$, $M = 1 + \varepsilon$, $0 < \varepsilon \ll 1$) on the flow of detonation products and, in particular, on the form of the limiting function $\mu_0(\varphi)$. It was found that with sufficiently small values of ε the parameters of the flow and the functions $\mu_0(\varphi)$ practically coincide with the corresponding values calculated in an exact statement. For example, with $\varepsilon = 0.04$, in a region containing 90-95% of the flow of mass of the detonation products, the maximal difference in the functions $\mu_0(\varphi)$ does not exceed 2% in both the plane and axisymmetric cases. Thus, at least to find the parameters of an asymptotic flow of detonation products, weakly supersonic boundary conditions can be used, which eliminates the need for cumbersome calculating procedures for the initial section.

As an illustration of the applicability of the asymptotic characteristics of a flow of detonation products, obtained above, let us consider the problem of the throwing of a sphere by a flat charge of explosive. We shall assume that the initial removal of the sphere from the surface of the charge is sufficiently great (10 thicknesses of the charge). The force acting on the sphere will be described by the Newton law

$$F(v)=\sigma(v)r_0^2\rho(\mathbf{U}-\mathbf{V})^2,$$

where r_0 is the radius of the sphere; ρ is the density of the detonation products; **V** is the velocity of the sphere; and $\sigma(v)$ is an experimentally determined function. In accordance with the existing experimental data [9], with fully established supersonic flow around the sphere $\sigma(v) = \text{const} = 1.436$. In the polar system of coordinates already used, using (12), the motion of the sphere is described by the system of equations

$$\frac{dV}{dR}=\frac{\sigma_0\mu_0(\varphi)}{RU}\frac{(U\cos(\Phi-\varphi)-V)}{V\cos(\Phi-\varphi)}(U^2+V^2)-2UV\cos(\Phi-\varphi)^{1/2};$$
$$d\Phi/dR=(\sigma_0\mu_0(\varphi)/RU)\,\mathrm{tg}(\varphi-\Phi)(U^2+V^2-2UV\cos(\Phi-\varphi))^{1/2};$$
$$d\varphi/dR=-\,\mathrm{tg}(\varphi-\Phi)/R,$$

where $\sigma_0 = 3\sigma(v)/(4\pi\rho_1 r_0)$; ρ_1 is the density of the material of the sphere; Φ is the angle of inclination of the vector of the velocity of the sphere to the Ox axis; and $\varphi(R)$ is the trajectory of the motion of the sphere. The initial data have the form $R = R_0$; $\varphi = \theta_1$; $V = D$; and $\Phi = 0$; D here is the detonation rate of the explosive, taken in dimensionless form; θ_1 is the slope of the limiting flow line of the detonation products.

The system of equations is invariant with respect to the extension of the coordinate R; therefore, all the solutions for a fixed value of σ_0 are similar:

$$V(R_0,\ R) = V(1,\ R/R_0),\ \varphi(R_0,\ R) = \varphi(1,\ R/R_0),$$

i.e., under the assumptions made, the result of the throwing does not depend on the initial position of the sphere R_0. The problem posed was solved numerically using the analytical dependence (16) for the detonation products of Hexogen with $\rho_0 = 1.66$ g/cm^3. To take account of the limited dimensions and of the distance at which the parameters of the sphere are fixed in the experiments, the calculation was made up to $R \simeq 5R_0$. The results of the calculation were compared with the results of experiments on the throwing of steel spheres by a flat charge of cast Trotyl and Hexogen 50/50 with the dimensions $110 \times 110 \times 20$ mm. The spheres (several each of two radii r_0) were arranged at a distance of 100 mm from the surface of the charge. The parameters of the accelerated spheres were determined by the result of an impact on a Duralium target, placed at a distance of 500 mm from the surface of the charge. In the experiments, measurements were made of the angles of the throwing and the depth of the cavity, using relationships given in [10]; these were used to calculate the velocity of the spheres. Figure 6 gives the calculated dependence of the velocity of the spheres V_n normal to the surface on the parameter σ_0 (solid curve); the figure also gives experimental data with the indicated scatter of the results, averaged over several experiments. A comparison showed satisfactory agreement between the experimental and calculated data.

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